Magnetohydrodynamic Stagnation Point Flow with a Convective Surface Boundary Condition

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This study analyzes the steady laminar two-dimensional stagnation point flow and heat transfer of an incompressible viscous fluid impinging normal to a horizontal plate, with the bottom surface of the plate heated by convection from a hot fluid. A uniform magnetic field is applied in a direction normal to the flat plate, with a free stream velocity varying linearly with the distance from the stagnation point. The governing partial differential equations are first transformed into ordinary differential equations, before being solved numerically. The analysis includes the effects of the magnetic parameter, the Prandtl number, and the convective parameter on the heat transfer rate at the surface. Results showed that the heat transfer rate at the surface increases with increasing values of these quantities.

Key words: Stagnation Point Flow; Magnetohydrodynamic; Convective Boundary Condition.

1. Introduction

The study of flow and heat transfer near a stagnation point has generated a lot of interests amongst researchers because there are many practical situations where fluids flowing impinging normally or obliquely to plane surfaces are encounted. Theories on the stagnation flow and associated heat transfer characteristics would be useful to enhance the technological developments involving related fields of study. Since the development of an exact solution for the two dimensional stagnation flow by Hiemenz [1] and an exact similar solution for the corresponding thermal field by Eckert [2], studies on the flow and heat transfer near a stagnation point has diversified to produce numerous results for stagnation-point flow and heat transfer, with different geometrical configurations, types of fluids, and boundary conditions. Stagnation point flows and related heat transfer problems are also encountered in problems involving stretching or shrinking sheets. Some examples of these studies can be found in [3 – 17]. More recently, Aziz [18], Magyari [19], Ishak [20], Ishak et al. [21], and Yao et al. [22] considered the similar problem for the case of convective boundary conditions. Aziz [18] considered the classical hydrodynamic and thermal boundary layers over a flat plate in a uniform stream of fluid and demonstrated that a similarity solution is possible if the convective heat transfer associated with the hot fluid on the lower surface of the plate is proportional to $x^{-1/2}$. Ishak [20] extended this study by introducing the effects of suction and injection on the flat surface, using the same assumption on the convective heat transfer coefficient at the plate's lower surface. Both Aziz [18] and Ishak [20] assumed a uniform free stream velocity.

The objective of the present study is to extend the work of Aziz [18] to include the effect of a uniform magnetic field applied in a direction normal to the flat plate, with a free stream velocity varying linearly with the distance x from the stagnation-point, i.e. $u_e(x) = ax$. The numerical analysis includes the effects of the magnetic parameter, the Prandtl number, and the convective parameter on the heat transfer rate at the surface.

2. Problem Formulation

Consider a steady laminar two-dimensional stagnation point flow of an incompressible viscous fluid impinging normal to a horizontal plate as shown in Figure 1. It is assumed that the free stream velocity is of the form $u_e(x) = ax$, where a is a constant. Further,

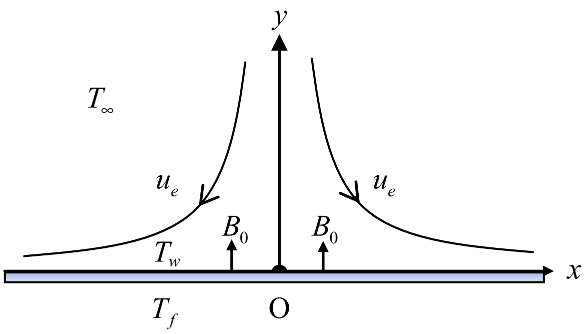

Fig. 1 (colour online). Physical model and coordinate system.

a uniform magnetic field of strength B_0 is assumed to be applied in the positive y-direction normal to the flat plate. The magnetic Reynolds number is assumed to be small, and thus the induced magnetic field is negligible. The boundary layer equations are [23 – 25]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_\mathrm{e}\frac{\mathrm{d}u_\mathrm{e}}{\mathrm{d}x}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0}{\rho}(u_\mathrm{e}-u), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the velocity components in the x and y-directions, respectively, T is the fluid temperature in the boundary layer, ν is the kinematic viscosity, and α is the thermal diffusivity. The boundary conditions for the flow field are

$$\begin{aligned}&u=0,\ \ v=0\ \text{ at }\ y=0,\\&u\to u_\mathrm{e}\ \text{ as }\ y\to\infty.\end{aligned} \tag{4}$$

It is assumed that the bottom surface of the plate is heated by convection from a hot fluid of temperature T_f which provides a heat transfer coefficient h_f.

Under this assumption, the boundary conditions for the thermal field may be written as [18]

$$\begin{aligned}&-k\frac{\partial T}{\partial y}=h_\mathrm{f}(T_\mathrm{f}-T_\mathrm{w})\ \text{ at }\ y=0,\\&T\to T_\infty\ \text{ as }\ y\to\infty,\end{aligned} \tag{5}$$

with k and T_w being the thermal conductivity and the uniform temperature over the top surface of the plate, respectively. Here we have $T_\mathrm{f}>T_\mathrm{w}>T_\infty$.

In order to solve (1) – (5), we introduce the following similarity transformation (see Aziz [18]):

$$\begin{aligned}&\eta=(u_\mathrm{e}/\nu x)^{1/2}y,\ \ \psi=(\nu x u_\mathrm{e})^{1/2}f(\eta),\\&\theta(\eta)=\frac{T-T_\infty}{T_\mathrm{f}-T_\infty}.\end{aligned} \tag{6}$$

In the above equations, η is the similarity variable, f is the dimensionless stream function, θ is the dimensionless temperature, and ψ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ which identically satisfies (1). Using (6) we obtain

$$u=axf'(\eta),\ \ v=-(\nu a)^{1/2}f(\eta), \tag{7}$$

where primes denote differentiation with respect to η. Substituting (6) and (7) into (2) and (3), we obtain

$$f'''+ff''+1-f'^2+M(1-f')=0, \tag{8}$$

$$\frac{1}{\mathrm{Pr}}\theta''+f\theta'=0, \tag{9}$$

where $\mathrm{Pr}=\nu/\alpha$ is the Prandtl number and M is the magnetic parameter defined as $M=\sigma B_0^2/(\rho a)$. The transformed boundary conditions are

$$\begin{aligned}&f(0)=0,\ \ f'(0)=0,\ \ -\theta'(0)=c[1-\theta(0)],\\&f'(\eta)\to 1,\ \ \theta(\eta)\to 0\ \text{ as }\ \eta\to\infty,\end{aligned} \tag{10}$$

where $c=(\nu/a)^{1/2}h_\mathrm{f}/k$ is the convective parameter. It should be mentioned that the free stream velocity $u_\mathrm{e}(x)$ is a function of x, hence it is different from the work of Aziz [18] and Ishak [20], in which a uniform free stream velocity was assumed. Due to this, in both their studies, it is required to assume that the heat transfer coefficient h_f is proportional to $x^{-1/2}$, in order for the similarity solution of the energy equation to exist. In the present study, due to the form of the free stream velocity, $u_\mathrm{e}(x)=ax$, this restriction is no longer necessary.

The quantities of physical interest are the values of $f''(0)$, being a measure of the skin friction, and the heat transfer rate at the surface $-\theta'(0)$. Our main aim is to investigate how the values of $f''(0)$ and $-\theta'(0)$ vary in terms of the parameters c, M, and Pr.

3. Results and Discussion

The ordinary differential equations (8) and (9) subject to the boundary conditions (10) were solved

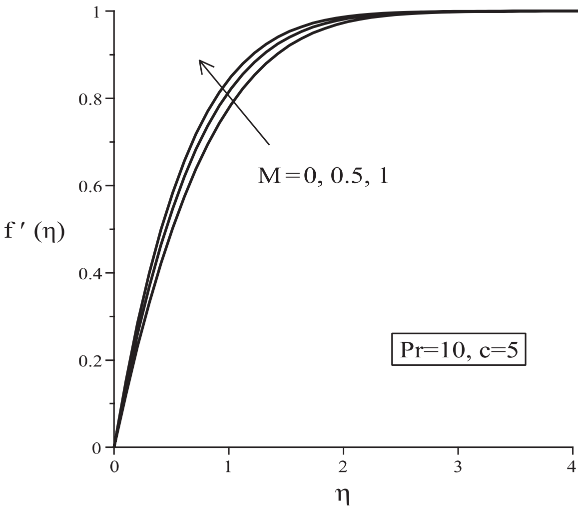

Fig. 2. Velocity profiles $f'(\eta)$ for various values of M when $\mathrm{Pr} = 10$ and $c = 5$.

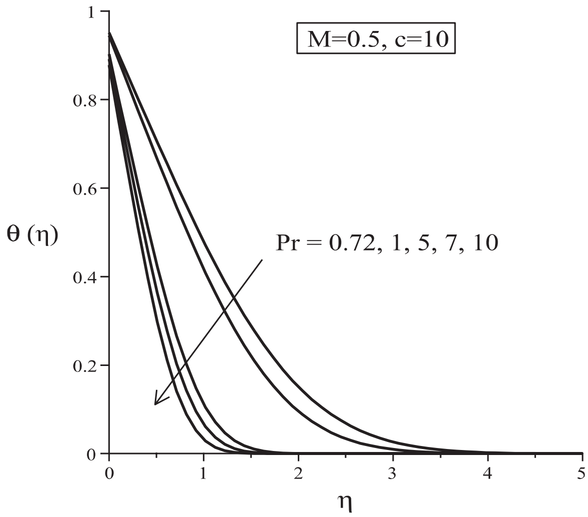

Fig. 3. Temperature profiles $\theta(\eta)$ for various values of Pr when $c = 10$ and $M = 0.5$.

numerically using a shooting method described in [15, 16]. Numerical solutions for the momentum equation (8) showed that the skin friction coefficient $f''(0)$ increases with the value of the magnetic parameter M, but is not affected by the values of either the convective parameter c or the Prandtl number Pr. This is expected since the momentum equation is independent of the parameters c and Pr. The velocity profiles presented in Figure 2 show that both the fluid velocity and the velocity gradient at the surface increase with the magnetic parameter M. This trend agrees with most earlier studies on the effect of magnetic field on the momentum transfer over a flat plate. Thus, the focus of the present study will be on the heat transfer problem. Table 1 presents the values of $-\theta'(0)$, which represents the heat transfer rate at the surface, for various values of the magnetic parameter M, the Prandtl number Pr, and the convective parameter c. As evident from Table 1, the values of $-\theta'(0)$ increase with the values of the convective parameter c, the magnetic parameter M, and the Prandtl number Pr. We notice that the values of $-\theta'(0)$ for the non-magnetic case $(M = 0)$ when $\mathrm{Pr} = 0.72$ are slightly higher than those given by Aziz [18] and Ishak [20] for the case of a uniform free stream. This is due to the fact that the present study assumes the free stream velocity to be a linear function of x, which accelerates the fluid motion in the boundary layer, and in consequence increases the surface shear stress and the heat transfer rate at the surface. Figure 3 shows the temperature profiles for various values of the Prandtl number Pr when the convective parameter and the magnetic parameter are fixed at $c = 10$ and $M = 0.5$, respectively. As evident from this figure, the surface temperature $\theta(0)$ decreases as

Table 1. Values of $-\theta'(0)$ for various values of c, M, and Pr.

c	$-\theta'(0)$								
	$M = 0$			$M = 0.5$			$M = 1$		
	$\mathrm{Pr} = 0.72$	$\mathrm{Pr} = 1$	$\mathrm{Pr} = 7$	$\mathrm{Pr} = 0.72$	$\mathrm{Pr} = 1$	$\mathrm{Pr} = 7$	$\mathrm{Pr} = 0.72$	$\mathrm{Pr} = 1$	$\mathrm{Pr} = 7$
0.05	0.045466	0.045971	0.047965	0.045557	0.046059	0.048030	0.045626	0.046126	0.048078
0.1	0.083373	0.085085	0.092178	0.083679	0.085388	0.092417	0.083912	0.085619	0.092598
0.2	0.142974	0.148083	0.170980	0.143876	0.149004	0.171805	0.144565	0.149707	0.172433
0.6	0.222505	0.292430	0.397568	0.276466	0.296043	0.402058	0.279022	0.298832	0.405514
0.8	0.273153	0.333005	0.476502	0.312459	0.337698	0.482966	0.315728	0.341333	0.487962
1	0.308235	0.363246	0.540942	0.338935	0.368837	0.549288	0.342785	0.373177	0.555760
5	0.33397	0.512045	0.953629	0.465026	0.523225	0.979874	0.472304	0.532001	1.000661
10	0.45573	0.539678	1.054156	0.487705	0.552113	1.086320	0.495716	0.561894	1.111927
20	0.477491	0.554645	1.112810	0.499895	0.567787	1.148713	0.508315	0.578137	1.177386

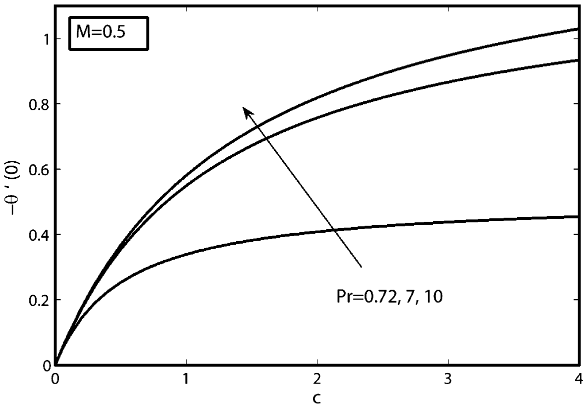

Fig. 4. The heat transfer rate at the surface $-\theta'(0)$ as a function of $c\,(>0)$ for various values of Pr when $M = 0.5$.

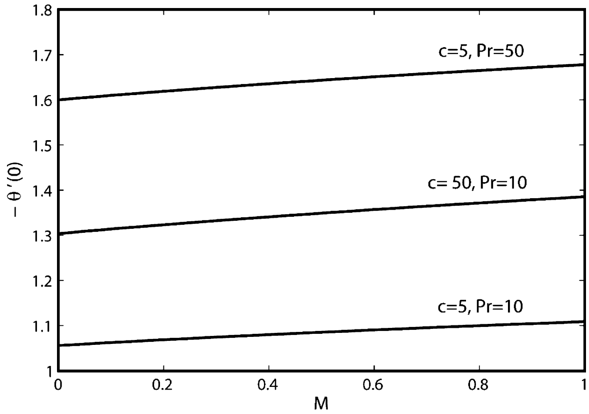

Fig. 6. The heat transfer rate at the surface $-\theta'(0)$ as a function of M for various values of c and Pr.

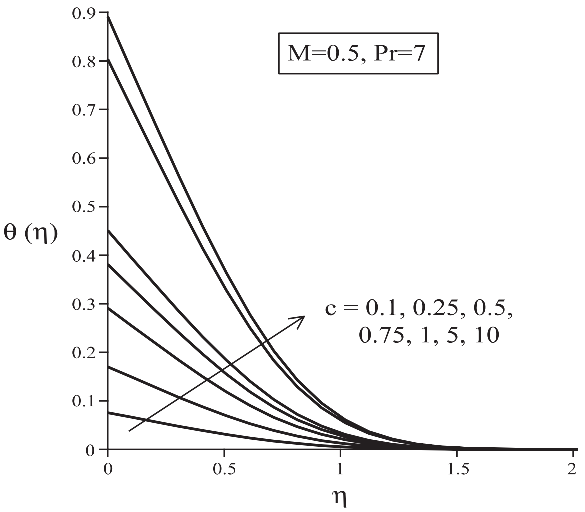

Fig. 5. Temperature profiles $\theta(\eta)$ for various values of $c\,(>0)$ when $\mathrm{Pr} = 7$ and $M = 0.5$.

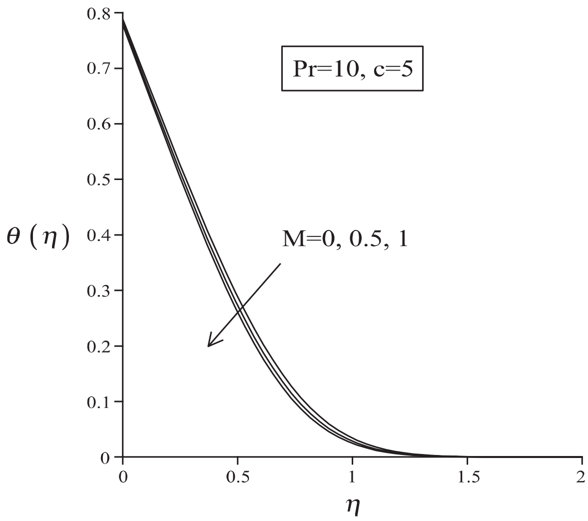

Fig. 7. Temperature profiles $\theta(\eta)$ for various values of M when $\mathrm{Pr} = 10$ and $c = 5$.

the Prandtl number Pr increases. Furthermore, as the value of Pr increases, the thermal boundary layer thickness decreases, resulting in an increase in the temperature gradient at the surface (in absolute sense). This trend can also be observed from Figure 4, which shows the variation of the heat transfer rate at the surface $-\theta'(0)$ with the convective parameter c for various values of Pr.

Figure 5 shows the variation of the temperature profiles with the convective parameter c when the magnetic parameter and the Prandtl number are fixed at $M = 0.5$ and $\mathrm{Pr} = 7$. The figure shows that the boundary layer thickens as c increases, and both the fluid temperature $\theta(\eta)$ and the heat transfer rate at the surface $-\theta'(0)$ increase with the convective parameter c.

Figure 6 presents the variation of $-\theta'(0)$ with the magnetic parameter M for various values of the Prandtl number Pr and the convective parameter c. The figure shows a slight increase in the heat transfer rate at the surface, (represented by the value of $-\theta'(0)$) as the magnetic parameter M increases. Figure 7 presents velocity profiles for various values of the magnetic parameter M for the case when $\mathrm{Pr} = 10$ and $c = 5$, indicating a slight decrease in the thermal boundary layer thickness and the fluid temperature as the value of the magnetic parameter increases.

4. Conclusions

We have investigated the fluid flow and heat transfer characteristics of a steady laminar two-dimensional stagnation point flow of an incompressible viscous fluid impinging normal to a horizontal plate, with the free stream velocity of the form $u_e(x) = ax$, and a uniform magnetic field of strength B_0 applied normal to the flat plate, with a convective boundary condition at the surface of the plate. It is found that the applied magnetic field increases the skin friction coefficient $f''(0)$, and consequently the surface shear stress. However, the Prandtl number Pr and the convective parameter c have no effect on the skin friction coefficient. Further, the heat transfer rate at the surface, $-\theta'(0)$, increases with the Prandtl number Pr, the convective parameter c, and the magnetic parameter M.

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